DETECTION OF RADAR SIGNALS IN NOISE

UNIT - 5

INTRODUCTION

- The two basic operations performed by radar are

1. detection of the presence of reflecting Objects

2. Extraction of information from the received waveform to obtain such target data as position, velocity, and perhaps size.

- Noise ultimately limits the capability of any radar.

MATCHED FILTER RECEIVER

- A network whose frequency-response function maximizes the output peak-signal-to-mean-noise (power) ratio is called a matched filter.

- The frequency-response function, denoted H(f), expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.

- If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio.

Contd…,

- If the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lowered signal-to-noise ratio.

- Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum.

- The receiver bandwidth B should be approximately equal to the reciprocal of the pulse width τ valid only for pulsed waveforms but not to other types of waveforms.

Contd…,

- The second detector and video portion of the well-designed radar super heterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.

- Narrow banding is most conveniently accomplished in the IF.

- For a received waveform s(t) with a given ratio of signal energy E to noise energy *No (or)* noise power per hertz of bandwidth, North showed that the frequency-response function of the linear, time-invariant filter.

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft)\, dt$ = voltage spectrum (Fourier transform) of input signal

$S^*(f)$ = complex conjugate of $S(f)$

t_1 = fixed value of time at which signal is observed to be maximum

G_a = constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be gaussian.

- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift exp (- j2∏ft$_l$). This phase shift varies uniformly with frequency.

Contd…,

- The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ and a phase spectrum exp [- $j\varphi_s(f)$].

- The matchedfilter frequency-response function may similarly be written in terms of its amplitude and phase spectra $|H(f)|$ and exp [-$j\varphi_m(f)$].

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_1]\}$$

or

$$|H(f)| = |S(f)|$$

and

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

Contd…,

- The amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

- The matched filter may also be specified by its impulse response h(t), which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi f t)\, df$$

$$h(t) = G_a \int_{-\infty}^{\infty} S^*(f) \exp[-j2\pi f(t_1 - t)]\, df$$

Contd…,

- Since S*(f) = S(-f), we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp\left[j2\pi f(t_1 - t)\right] df = G_a s(t_1 - t)$$

- the impulse response of the matched filter is the image of the received waveform.

Matched filter Characteristics and Correlation function

- The output of the matched filter is not a replica of the input signal
- The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t1.
- The crosscorrelation function R(t) of two signals y(λ) and s(λ), each of finite duration, is defined as

$$R(t) = \int_{-\infty}^{\infty} y(\lambda) s(\lambda - t)\, d\lambda$$

- The output $y_o(t)$ of a filter with impulse response h(t) when the input is $y_{in}(t) = s(t) + n(t)$ is

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) h(t - \lambda)\, d\lambda$$

- If the filter is a matched filter, then $h(\lambda) = s(t_1 - \lambda)$ and Eq. above becomes

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) s(t_1 - t + \lambda)\, d\lambda = R(t - t_1)$$

- Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal.
- The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function.

- If the input signal $y_{in}(t)$ were the same as the signal s(t) for which the matched filter was designed, the output would be the autocorrelation function.
- The autocorrelation function of a rectangular pulse of width τ is a triangle whose base is of width 2τ.

DETECTION CRITERIA

- The detection of weak signals in the presence of noise is equivalent to deciding whether the receiver output is due to noise alone or to signal-plus-noise.
- This is the type of decision probably made (subconsciously) by a human operator on the basis of the information present at the radar indicator.
- When the detection process is carried out automatically by electronic means the radar detection process was described in terms of threshold detection.

- There are two types of errors that might be made in the decision process.

- This occurs whenever the noise is large enough to exceed the threshold level. In statistical detection theory it is sometimes called a type I error. The radar engineer would call it a *false alarm*.

- *A type II error is one in which the* signal is erroneously considered to be noise when signal is actually present. This is a *missed* detection to the radar engineer.

- The setting of the threshold represents a compromise between these two types of errors.

- Neyman-Pearson observer:

- Most radars utilize the equivalent of the Neyman-Pearson Observer and operate with a fixed number of pulses.

- The threshold level was selected so as not to exceed a specified false-alarm probability; that is, the probability of detection was maximized for a fixed probability of false alarm.

- This is equivalent to fixing the probability of a type I error and minimizing the type II error.

- This type of threshold detector is sometimes called a ***Neyman-Pearson Observer.***

- Likelihood-ratio receiver:

- The likelihood ratio is an important statistical tool and may be defined as the ratio of the probability-density function corresponding to signal-plus-noise, $P_{sn}(v)$,*to the probability-density function of noise alone*, $p_n(v)$.

$$L_r(v) = \frac{p_{sn}(v)}{p_n(v)}$$

- It is a measure of how likely it is that the receiver envelope ***v is due to signal-plus-noise as*** compared with noise alone.

- If the likelihood ratio $L_r(v)$ *is sufficiently large, it would be reasonable to conclude that the signal* was indeed present.

- Thus detection may be accomplished by establishing a threshold at the out put of a receiver which computes the likelihood ratio.

- **Inverse probability receiver:**

- This method has been applied by Woodward and Davies to the reception of signals in noise.

- It is based upon the application of Bayes' rule for the probability of causes.

- The problem of forming the best estimate of the cause of the event is a problem in inverse probability.
- A limitation of the method of inverse probability based on the application of Bayes' rule is the difficulty of specifying the a priori probabilities.
- whenever the a priori probabilities are known, the inverse-probability method may be used with confidence.
- When the a priori probabilities are not known, the likelihood-ratio test is usually employed.

- **Relationship of inverse probability and likelihood-ratio receivers:**

$$p(SN \mid y) = \frac{L_r(y)p(SN)}{L_r(y)p(SN) + 1 - p(SN)}$$

- The likelihood ratio follows from inverse probability if the assumption is made that the a priori probabilities are equally likely.

- Both the methods may be implemented by computing the cross-correlation function between the received signal and the signal $\boldsymbol{s_i(t)}$.

- **Ideal observer:**

- Idea Observer was formulated by Siegert.

- The criterion of the Ideal Observer maximizes the total probability of a correct decision (or) minimizes the total probability of an error.

- The probability of an error is

$$p(E) = p(N)p_{fa} + p(SN)p_m$$

where $p(N)$ and $p(SN)$ = a priori probabilities of obtaining noise and signal-plus-noise, respectively $[p(N) + p(SN) = 1]$

p_{fa} = conditional probability of a false alarm

p_m = conditional probability of a miss,

- These probabilities must be known beforehand in the Ideal Observer.
- The Ideal Observer applies equal weight to an error due to a false alarm and to an error due to a miss.
- If the errors are not of equal importance, the theory of the Ideal Observer may be modified to take this into account using the concepts of ***statistical decision theory***

- **Sequential observer:**

- Sequential *Observer* makes an observation of the receiver output and, on the basis of this single observation, decides between one of three choices:

(1)The receiver output is due to the presence of signal with noise

(2)The output is due to noise alone; or

(3)The available evidence is not convincing enough to make a decision between choices 1 and 2

- The term sequential detection is sometimes used synonymously with Sequential Observer.

- Sequential detection has also been used to describe a two-step that can be employed with phased-array radar.

DETECTOR CHARACTERISTICS

- The portion of the radar receiver which extracts the modulation from the carrier is called the detector.
- One form of detector is the envelope detector, which recognizes the presence of the signal on the basis of the amplitude of the carrier envelope. All phase information is destroyed.
- The zerocrossings detector destroys amplitude information.
- The coherent detector is an example of one which uses both phase and amplitude.

- **Envelope detector:**
- The function of the envelope detector is to extract the modulation and reject the carrier. By eliminating the carrier, all phase information is lost and the detection decision is based solely on the envelope amplitude.

- The envelope detector consists of a rectifying element and a low-pass filter to pass the modulation frequencies but to remove the carrier frequency.

- In the linear detector the output signal is directly proportional to the input envelope.

- in the square-law detector, the output signal is proportional to the square of the input envelope.

- Assume that there are n independent pulses with envelope amplitudes v_1, v_2, . . . , on available from the radar receiver.

- The probability-density function for the envelope of 11 independent noise samples is the product of the probability-density function for each sample.

$$p_n(n, v_i) = \prod_{i=1}^{n} p_n(v_i)$$

The probability-density function for ith noise pulse $p_n(v_i)$ is given by

$$p_n(v_i) = v_i \exp\left(-\frac{v_i^2}{2}\right)$$

where r_i is the ratio of the envelope amplitude R to the rms noise voltage $\psi_0^{1/2}$.

probability-density function for the envelope of n signal-plus-noise pulses is

$$p_s(n, v_i) = \prod_{i=1}^{n} p_s(v_i)$$

The probability-density function for signal-plus-noise, $p_s(v_i)$, is the Rice distribution

$$p_s(v_i) = v_i \exp\left[-\frac{(v_i^2 + a^2)}{2}\right] I_0(av_i)$$

a = ratio of signal (sine-wave) amplitude to rms noise voltage

$I_0(x)$ = modified Bessel function of zero order.

- **Logarithmic detector:**

- If the output of the receiver is proportional to the logarithm of the input envelope, it is called a *logarithmic receiver.*

- The detection characteristics (probability of detection as a function of the probability of false alarm, signal-to-noise ratio, and the number of hits integrated) for the logarithmic receiver have been computed by Green, following the methods of Marcum.

- It might be used to prevent receiver saturation or to reduce the effects of unwanted clutter targets in certain types of non-MTI radar receivers

- **Zero-crossings detector:**
- The information contained in the zero crossings of the received signal can, be used for detecting the presence of signals in noise.

- The greater the signal-to-noise ratio the less will be the average number of zero crossings.

- The average number of zero-crossings per second at the output of a narrow-bandpass filter of rectangular shape when the input is a sine wave in gaussian noise is

$$\bar{n}_0 = 2f_0\left[\frac{S/N + 1 + (f_B^2/12f_0^2)}{S/N + 1}\right]^{1/2}$$

- **Coherent detector:**

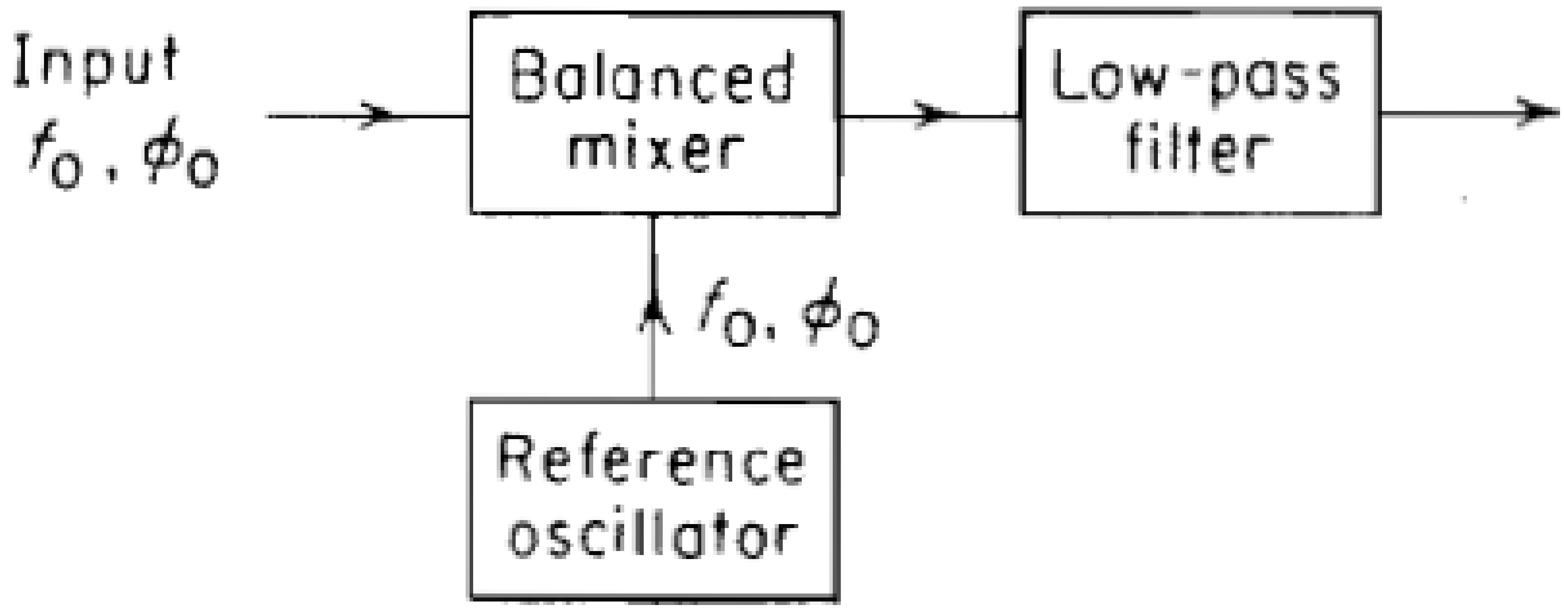


Figure Basic configuration of a coherent detector.

- The coherent detector consists of a reference oscillator feeding a balanced mixer.

- The input to the mixer is a signal of known frequency f, and known phase ϕ_0 plus its accompanying noise.

- The reference-oscillator signal is assumed to have the same frequency and phase as the input signal to be detected.

- The output of the mixer is followed by a low-pass filter which allows only the d-c and the low-frequency modulation components to pass while rejecting the higher frequencies in the vicinity of the carrier.

- Although the coherent detector may be of superior sensitivity than other detectors it is seldom used in radar applications since the phase of the received signal is not usually known.

AUTOMATIC DETECTION

- The function of the radar operator viewing the ordinary radar display is to recognize the presence of targets and extract their location.

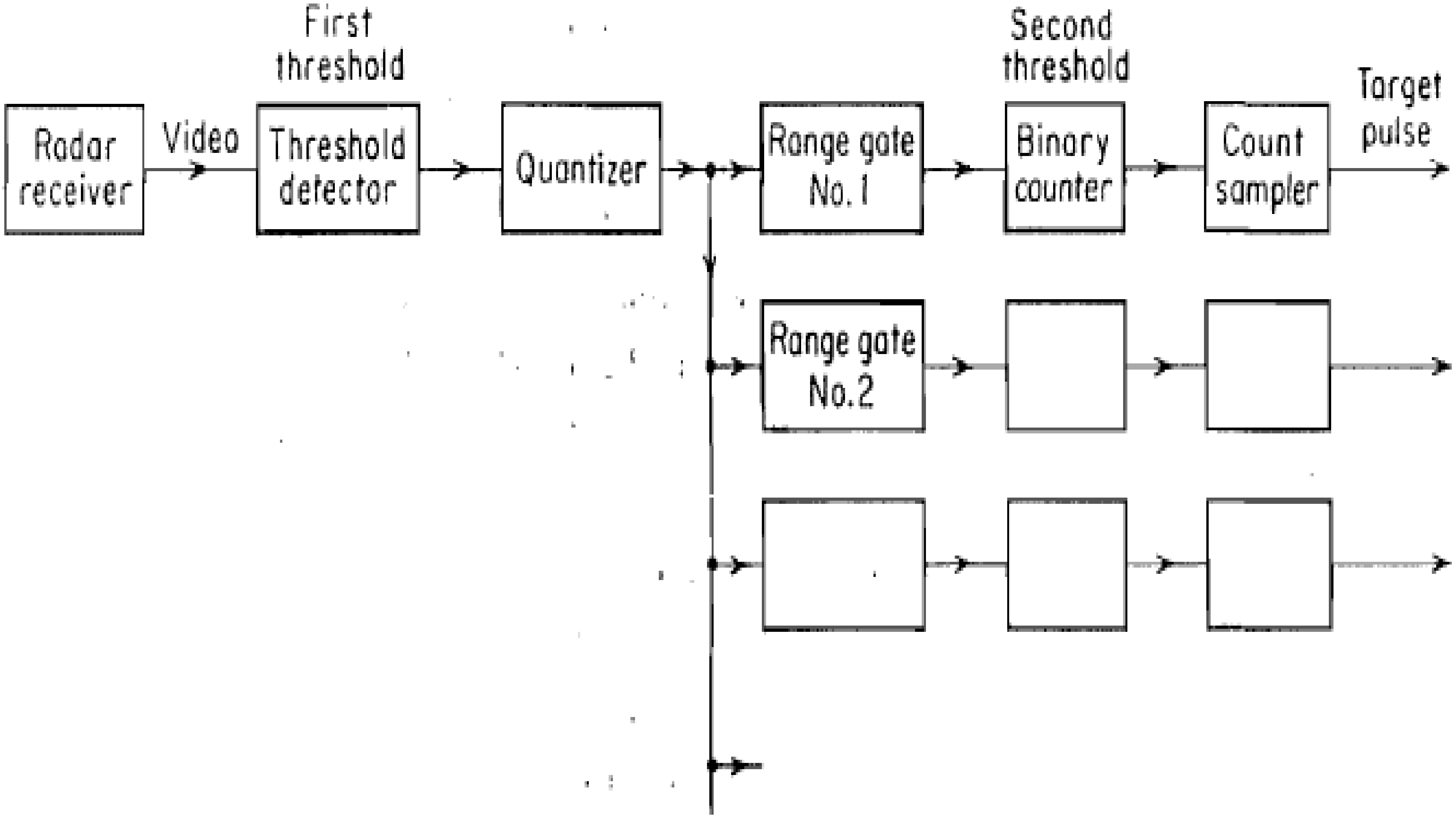
- When the function is performed by electronic decision circuitry without the intervention of an operator, the process is known as ***automatic detection.***

- The automatic detector has also been called ***plot extractor and data extractor.***

- Usually there are four basic aspects to automatic detection:

(1) The integration of the pulses received from the target.
(2) The detection decision; and the determination of the target location in.
(3) Range.
(4) Azimuth.

- **Binary moving-window detector:**



Block diagram of a binary moving window detector, or binary integrator.

- it has also been called double-threshold detector, m-out-of-n detector, coincidence detector, sliding-window detector, and binary integrator.

- it has the advantage of simplicity.

- The radar video is passed through a threshold detector, which may be thought of as a bottom clipper.

- Only those signals whose amplitude exceeds the preset threshold are allowed to pass.

- This is the first of two thresholds, hence the name double-threshold detector.
- The output of the first threshold is sampled by the quantizer at least once per range-resolution cell.
- A standard pulse is generated if the video waveform exceeds the first threshold, and nothing if it does not
-
- These are designated by 1 or 0 respectively.
- Thus the output of the quantizer is a series of 1's and 0's.
- This is then separated into separate range cells by the range gate.

the 1s and 0s from the last n sweeps are stored and counted in the binary counter. there are at least m 1s within the last n sweeps, a target is said to be present.

- The number m is the second threshold to be passed in the double-threshold detector. The two thresholds must be selected jointly for best performance.
- Advantages:

1. it is less sensitive to the effects of a single large interference pulse that might exist along with the target echo pulses.
2. An analysis of the problem of detection of signals in clutter

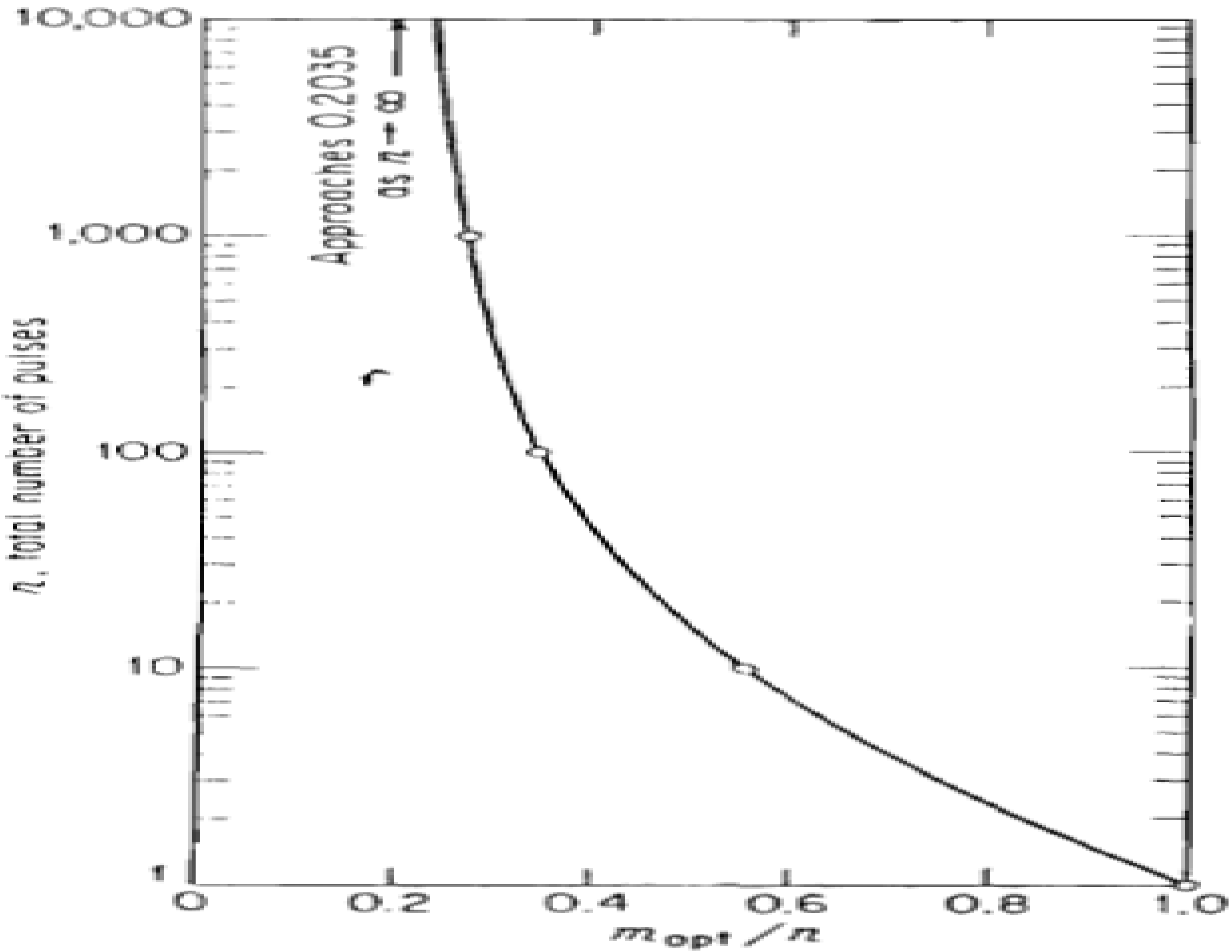


Figure: Optimum number of pulses m $_{opt}$ (out of a maximum of n) for a binary moving window detector. A constant (nonfluctuating) target is assumed.

- **Tappeddelay-line integrator:**

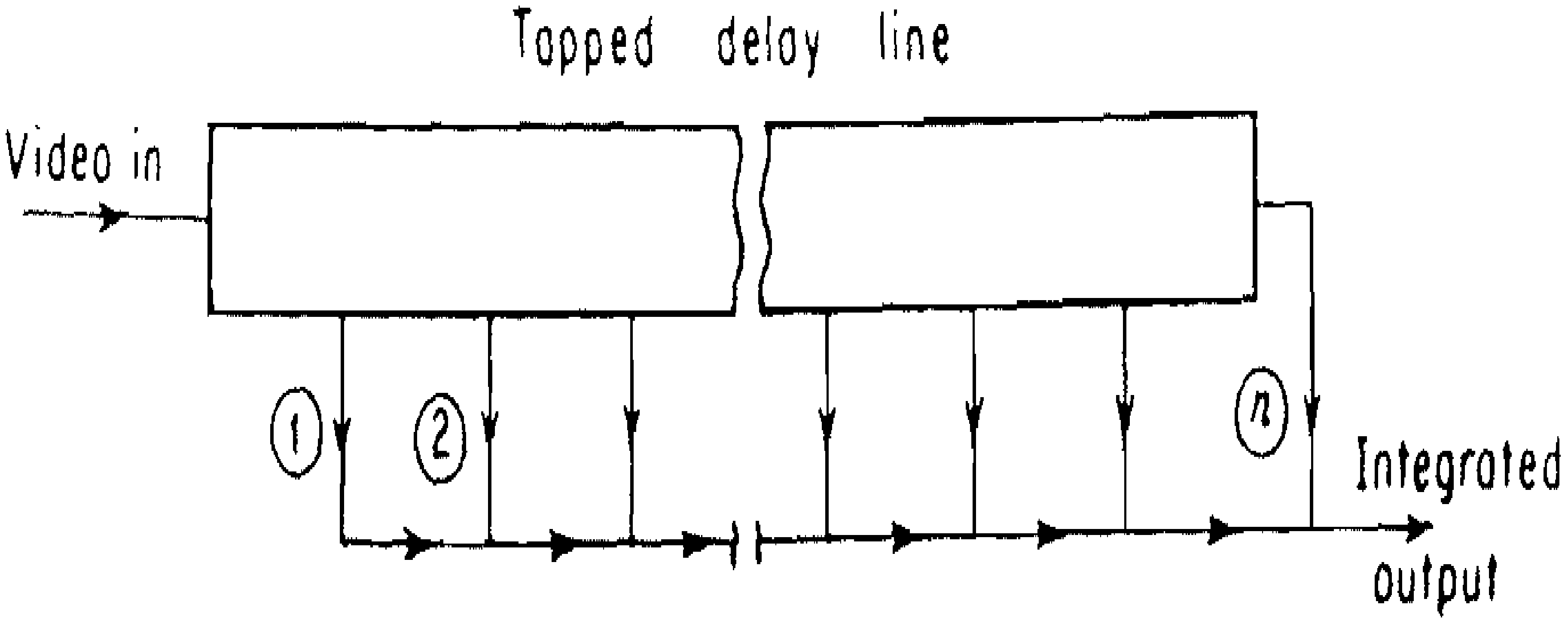


Figure: Pulse integrator using tapped delay line.

- This form of integrator is also known as the moving-window detector or analog moving-window detector.

- The time delay through the line is made equal to the total integration time, and the taps are spaced at intervals equal to the pulse-repetition period.

- The number of taps equals the number of pulses to be integrated.

- The outputs from each of the taps are tied together to form the sum of the previous n pulses.

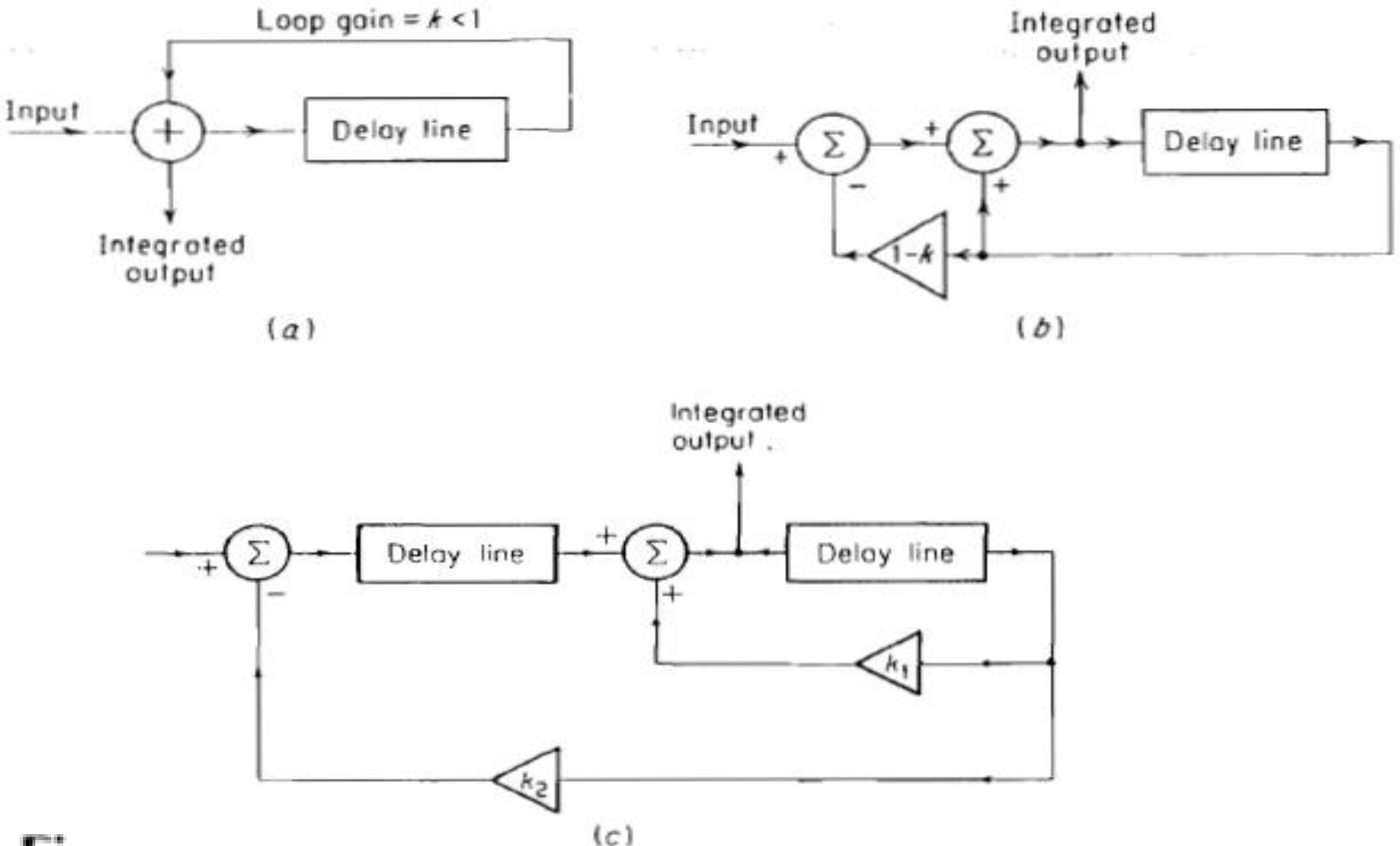
- It is similar to the binary moving-window detector.
- It does not suffer the 1.5 to 2 dB loss of the binary detector.
- It can be employed to estimate the target's angle by beam splitting.
- The angular location of the target can be estimated from the output of this detector by taking the midpoint between the first and last crossings of the detection threshold, or by taking the maximum value of the running sum.
- The accuracy of the location measurement is only about 20 percent worse than theoretical.

- **Recirculating-delay-line integrator:**

- The delay-line integrator, or moving-window detector, described above requires that a number of pulses, equal to that expected from the target, be held in storage.

- A simplification can be had by recirculating the output through a single delay line whose delay is equal to the pulse repetition period.

- The recirculating delay-line integrator is also called as feedback integrator, adds each new sweep to the sum of all the previous sweeps



Figure

Recirculating-delay-line integrator, or feedback integrator, k = loop gain < 1. (a) single delay loop; (h) double loop: (c) two-pole filter.

- To prevent unwanted oscillations, or "ringing," due to positive feedback, the sum must be attenuated by an amount k after each pass through the line.

- The factor k is the gain of the loop formed by the delay line and the feedback path.

- It must be less than unity for stable operation. The effect of $k < 1$ is that the integrator has imperfect "memory."

- The optimum value of k depends on the number of pulses received from the target.

- The factor k in the recirculating-delay-line integrator of Fig. a is equivalent to an exponential weighting of the received pulses.

- It results in a loss of about 1.0 dB in signal-to-noise ratio as compared with the ideal postdetection integrator that weights the received pulses in direct proportion to the fourth power of the antenna beam pattern.

- It is 0.5 dB less efficient than the moving window detector with uniform weights.

- The double-loop integrator of Fig.b is a two-pole filter with a multiple pole.

- It is about 0.3 dB less efficient than the ideal integrator with optimum weights.

- The double delay-tine configuration of Fig. c is also a two-pole filter but unlike the double-loop integrator, the two poles need not be at the same location.

- Its detection performance is only 0.15 dB less efficient than the optimum.

- The recirculating delay-line integrators can be used to obtain an estimate of the angular location of the target.

- The target can be found, as in beam splitting, by taking the midpoint between the start and end of the threshold crossing.

- Its relatively good angle-estimating accuracy, good detection performance, along with the relative simplicity of a feedback integrator makes the two-pole filter a good choice as an automatic detector for scanning radars.

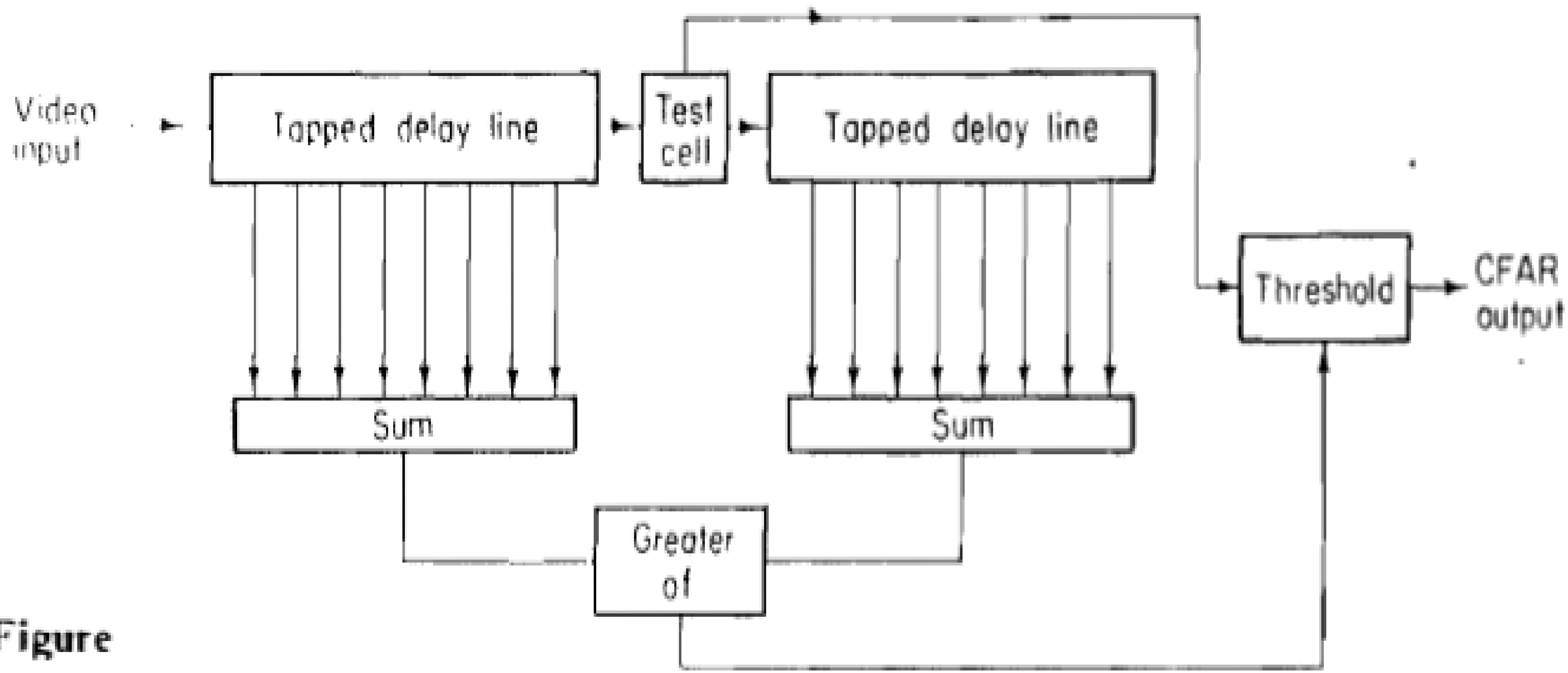
CONSTANT-FALSE-ALA RM-RATE (CFAR) RECEIVER

- The threshold at the output of a radar receiver, is chosen so as to achieve a desired false-alarm probability.

- The false-alarm rate is quite sensitive to the threshold level.

- If changes in the false-alarm rate are gradual, an operator viewing a display can compensate with a manual gain adjustment.

- A CFAR may be obtained by observing the noise or clutter background in the vicinity of the target and adjusting the threshold in accordance with the measured background.



Figure

Cell averaging CFAR. In this version the greater of the outputs from the range cells ahead of or behind the cell of interest is used to set the threshold.

- The output of the test cell is the radar output.

- The spacing between the taps is equal to the range resolution.

- The outputs from the delay line taps are summed. This sum, when multiplied by the appropriate constant, determines the threshold level for achieving the desired probability of false alarm.

- Thus the threshold varies continuously according to the noise or the clutter environment found within a range interval surrounding the range cell under observation.

- This form of CFAR has sometimes been called Adaptive Video Threshold, or AVT.

- The threshold is determined by whichever of the two sums is the greater.

- Typically the number of taps used in a cell-averaging CFAR might vary from 16 to 20.

- If the target echo is large, energy can spill over into the adjacent range-resolution cells and affect the measurement of the average background.

- Fur this reason, the range cells surrounding the test cell are often omitted when averaging the background.

- There are several other methods for achieving CFAR besides the use of cell averaging.

1. One of the first CFAR receivers used is postdetection integrator to estimate the average noise.

2. Another CFAR technique is the hard limiter, sometimes called the Dicke fix.

- Advantage: CFAR is widely used to prevent clutter and noise interference from saturating the display of an ordinary radar and preventing targets from being obscured.

- Disadvantages:

- It introduces an additional loss compared to optimum detection, and in some systems the number of pulses processed needs to be large to keep the loss low.

- CFAR maintains the false-alarm rate constant at the expense of the probability of detection. Thus, it causes targets to be missed.

- The operator is usually given no indication that there may be missed detections.

Efficiency of non matched filters

- In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of non matched filters compared with the ideal matched filter.
- The measure of efficiency is taken as the peak signal-to noise ratio from the non matched filter divided by the peak signal-to-noise ratio (2E/No) from the matched filter.
- It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

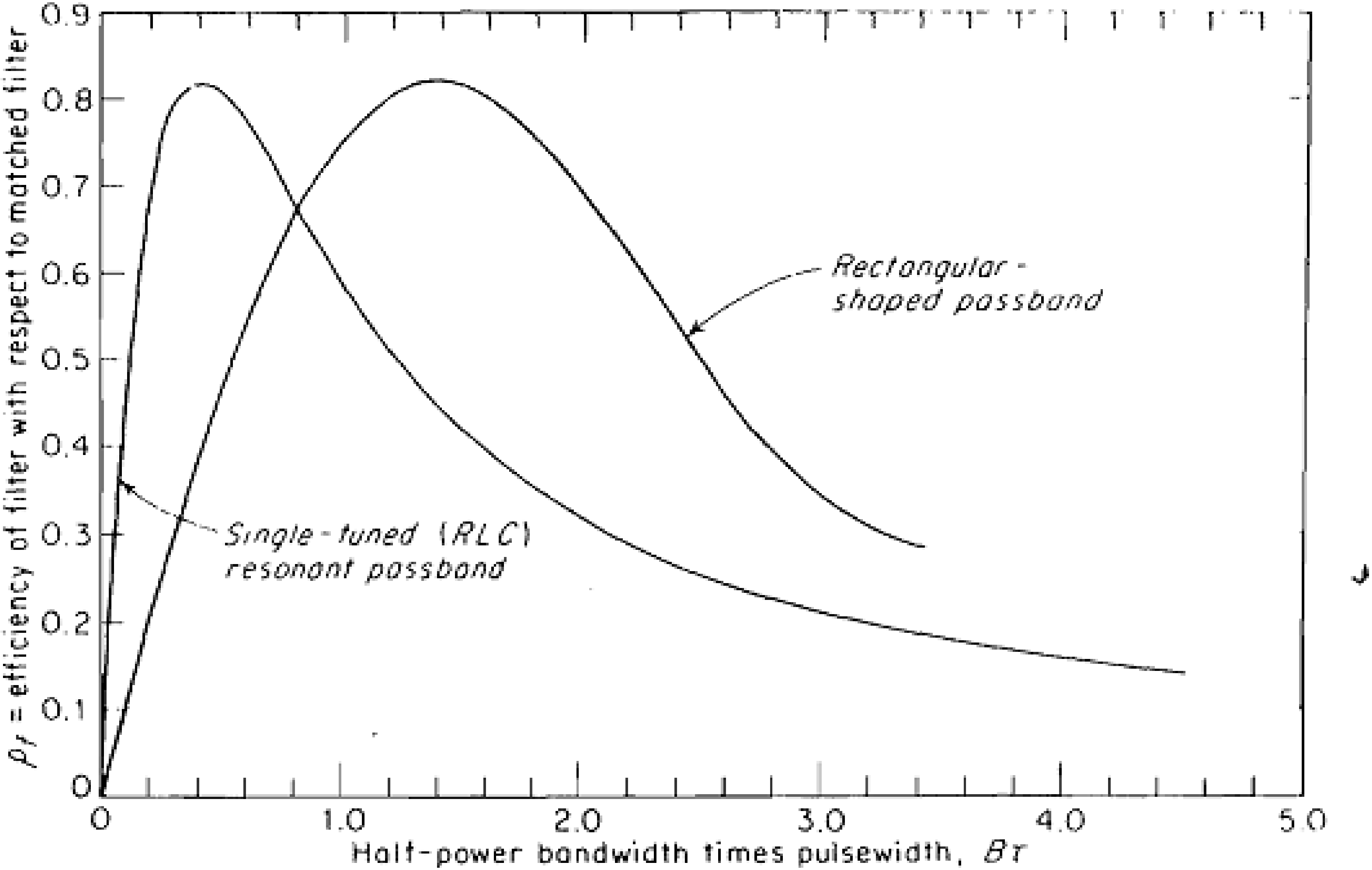


Fig.1 Efficiency, relative to a matched filter, of a single-tuned resonant filter and a rectangular shaped filter, when the input signal is a rectangular pulse of width τ. B –filter bandwidth.

Input signal	Filter	Optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

Table 1 Efficiency of non-matched filters compared with the matched filter

Matched filter with Non-White noise

- In the derivation of the matched-filter characteristic, the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency.
- If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.
- It has been shown that if the input power spectrum of the interfering noise is given by $[N_i\ (f)]^2$, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

- When the noise is non-white, the filter which maximizes the output signal-to-noise ratio is called the NWN (non-white noise) matched filter.
- For white noise $[N_i(f)]^2$ = constant and the NWN matched-filler frequency-response function of Eq. above reduces to that of Eq. discussed earlier in white noise. Equation above can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right)^* \exp(-j2\pi f t_1)$$

- This indicates that the NWN matched filter can be considered as the cascade of two filters.
- The first filter, with frequency-response function $l/N_i(f)$, acts to make the noise spectrum uniform, or white.
- It is sometimes called the whitening filter.
- The second is the matched filter when the input is white noise and a signal whose spectrum is $S(f)/Ni(f)$.